Quiz 2 Solutions

1. Evaluate the following integrals:

(a) $\int x^2 5^x \, dx$

$$\int x^2 5^x \, dx = \frac{1}{\ln(5)} x^2 5^x - \int \frac{1}{\ln(5)} \cdot 2x \, 5^x \, dx$$

$$\boxed{\begin{array}{ll} f(x) = x^2 & g(x) = \frac{1}{\ln(5)} 5^x \\ f'(x) = 2x & g'(x) = 5^x \end{array}}$$

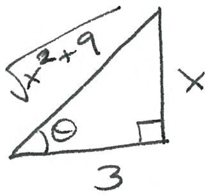
$$= \frac{1}{\ln(5)} x^2 5^x - \frac{2}{\ln(5)} \int x \, 5^x \, dx$$

$$\boxed{\begin{array}{ll} f_1(x) = x & g_1(x) = \frac{1}{\ln(5)} 5^x \\ f_1'(x) = 1 & g_1'(x) = 5^x \end{array}}$$

$$= \frac{1}{\ln(5)} x^2 5^x - \frac{2}{\ln(5)} \left(\frac{1}{\ln(5)} x \, 5^x - \int \frac{1}{\ln(5)} 5^x \, dx \right)$$

$$= \boxed{\frac{1}{\ln(5)} x^2 5^x - \frac{2}{\ln(5)} \left(\frac{1}{\ln(5)} x \, 5^x - \frac{1}{\ln(5)} \cdot \frac{1}{\ln(5)} 5^x \right) + C}$$

(b) $\displaystyle\int \frac{\sqrt{2x^2+18}}{x^4}\,dx$.



$\tan\theta = \frac{x}{3}$

$3\tan\theta = x$

$dx = 3(\sec\theta)^2\,d\theta$

$$\int \frac{\sqrt{2x^2+18}}{x^4}\,dx = \sqrt{2}\int \frac{\sqrt{x^2+9}}{x^4}\,dx$$

$$= \sqrt{2}\int \frac{\sqrt{(3\tan\theta)^2+9}}{(3\tan\theta)^4}\left(3(\sec\theta)^2\right)d\theta$$

$$= 3\sqrt{2}\int \frac{\sqrt{(\tan\theta)^2+1}}{3^3(\tan\theta)^4}(\sec\theta)^2\,d\theta$$

$$= \frac{3\sqrt{2}}{3^3}\int \frac{(\sec\theta)^3}{\cancel{3}\,(\tan\theta)^4}\,d\theta$$

$$= \frac{\sqrt{2}}{3^2}\int \frac{\cos\theta}{(\sin\theta)^4}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{\sqrt{2}}{3^2}\int \frac{1}{u^4}\,d\theta$$

$$= \frac{\sqrt{2}}{3^2}\left(\frac{1}{-3}\right)\cdot\left(u^{-3}\right) + C$$

$$= -\frac{\sqrt{2}}{3^3}\cdot\frac{1}{(\sin\theta)^3} + C$$

$$\boxed{= \frac{\sqrt{2}}{3^3}\cdot\left(\frac{\sqrt{x^2+9}}{x}\right)^3 + C}$$

(c) $\int \arcsin(x)\,dx$

$$\int \arcsin(x)\,dx = x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$u = \arcsin(x) \qquad v = x$

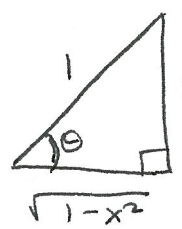
$du = \frac{1}{\sqrt{1-x^2}}\,dx \qquad dv = dx$

$$= x\arcsin(x) - \int \frac{\sin\theta}{\sqrt{1-(\sin\theta)^2}}\cos\theta\,d\theta$$

$$= x\arcsin(x) - \int \sin\theta\,d\theta$$

$$= x\arcsin(x) + \cos\theta + C$$

$$= \boxed{x\arcsin(x) + \sqrt{1-x^2} + C}$$



$\sin\theta = x$

$dx = \cos\theta\,d\theta$

(d) $\int \frac{r^2}{r+4}\,dr$

$$\begin{array}{r} r-4 \\ r+4 \overline{)\,r^2 \qquad\quad} \\ -(r^2+4r) \\ \hline -4r \\ -(-4r-16) \\ \hline 16 \end{array}$$

$$\int \frac{r^2}{r+4}\,dr = \int r-4 + \frac{16}{r+4}\,dr$$

$$= \boxed{r^2-4r + 16\ln|r+4| + C}$$

(e) $\int_0^{\pi} e^{\cos(t)} \sin(2t)\, dt$

$$\int e^{\cos(t)} \sin(2t)\,dt = \int e^{\cos(t)} \left(2 \sin(t) \cos(t)\right) dt$$

$$= 2 \int e^{\cos(t)} \sin(t) \cos(t)\, dt$$

$$\boxed{\begin{aligned} u &= \cos(t) \\ du &= -\sin(t) \end{aligned}}$$

$$= -2 \int e^{u}\, u\, du$$

$$\boxed{\begin{aligned} f(x) &= u & g(x) &= e^u \\ f'(x) &= 1 & g'(x) &= e^u \end{aligned}}$$

$$= -2 \left(u e^{u} - \int e^{u}\, du \right)$$

$$= -2 \left(u e^{u} - e^{u} \right) + C$$

$$= \boxed{-2 \left(\cos(t) e^{\cos(t)} - e^{\cos(t)} \right) + C}$$

(f) $\int_0^{1} \frac{x-4}{x^2-5x+6}\, dx$

$$\int \frac{x-4}{x^2-5x+6}\, dx = \int \frac{x-4}{(x-3)(x-2)}\, dx = \int \frac{-1}{x-3}\, dx + \int \frac{2}{x-2}\, dx$$

$$= -\ln|x-3| + 2\ln|x-2| + C$$

$$\frac{x-4}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2} = \frac{-1}{x-3} + \frac{2}{x-2}$$

$$x-4 = A(x-2) + B(x-3)$$

$$x-4 = Ax - 2A + Bx - 3B$$

$x: \; 1 = A + B \;\rightarrow\; B = 1 - A$

$c: \; -4 = -2A - 3B \;\rightarrow\; -4 = -2A - 3(1-A)$

$-4 = -2A - 3 + 3A$

$-4 = A - 3$

$-1 = A$

$B = 1 - (-1)$

$B = 2$

So $\int_0^{1} \frac{x-4}{x^2-5x+6}\, dx = -\ln|x-3| + 2\ln|x-2|$

$$= \left(-\ln|1-2| + 2\ln|1-2|\right)$$

$$- \left(-\ln|0-3| + 2\ln|0-$$

$$= -\ln(1) + 2\ln(1)$$

$$+ \ln|3| - 2\ln|2|$$

$$= \boxed{\ln(3) - 2\ln(2)}$$

(g) $\int \theta \sec\theta \tan\theta \, d\theta$.

$$\int \theta \sec\theta \tan\theta \, d\theta = \theta\sec\theta - \int \sec\theta \, d\theta$$

$$= \boxed{\theta\sec\theta - \ln|\sec\theta + \tan\theta| + C}$$

$$\boxed{\begin{array}{ll} u = \theta & v = \sec\theta \\ du = d\theta & dv = \sec\theta\tan\theta\, d\theta \end{array}}$$

(h) $\int \frac{x^2 - x + 6}{x^3 + 3x} dx$

$$\frac{x^2 - x + 6}{x^3 + 3x} = \frac{(x-3)(x+2)}{x(x^2+3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3} = \frac{2}{x} + \frac{-x-1}{x^2+3}$$

$$= \frac{2}{x} - \frac{x+1}{x^2+3}$$

$$x^2 - x + 6 = (x-3)(\cancel{x+2}) = A(x^2 + 3) + (Bx + C)x$$

$$x^2 - x + 6 = Ax^2 + 3A + Bx^2 + Cx$$

x^2: $1 = A + B$ $\rightarrow$ $1 = 2 + B$, $B = -1$

x: $-1 = C$ $\rightsquigarrow$ $C = -1$

c: $6 = 3A$ $\rightsquigarrow$ $A = 2$

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx = \int \frac{2}{x} - \frac{x+1}{x^2+3} \, dx = 2\ln|x| - \left(\int \frac{x}{x^2+3} + \frac{1}{x^2+3} \, dx\right)$$

$$= 2\ln|x| - \underbrace{\int \frac{x}{x^2+3} dx}_{u = x^2+3,\ du = 2x\,dx} - \int \frac{1}{x^2+3} dx$$

$$= 2\ln|x| - \frac{1}{2}\int \frac{1}{u} du - \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right)$$

$$= 2\ln|x| - \frac{1}{2}\ln|u| - \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

$$= \boxed{2\ln|x| - \frac{1}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C}$$

(i) $\int (\tan(x))^5 (\sec(x))^4 \, dx$

$$\int (\tan(x))^5 (\sec(x))^4 \, dx = \int (\tan(x))^5 (\sec(x))^2 (\sec(x))^2 \, dx$$

$$= \int (\tan(x))^5 \left((\tan(x))^2 + 1\right) (\sec(x))^2 \, dx$$

$u = \tan(x)$
$du = (\sec(x))^2 \, dx$

$$= \int u^5 (u^2 + 1) \, du$$

$$= \int u^7 + u^5 \, du$$

$$= \frac{u^8}{8} + \frac{u^6}{6} + C$$

$$= \boxed{\frac{(\tan(x))^8}{8} + \frac{(\tan(x))^6}{6} + C}$$

(j) $$\int_0^{3/5} \sqrt{9 - 25x^2} \, dx = \int_{x=0}^{x=\frac{3}{5}} \sqrt{9 - 25\left(\frac{3\sin\theta}{5}\right)^2} \cdot \frac{3}{5}\cos\theta \, d\theta$$

$$= \frac{3^2}{5} \int_0^{\pi/2} \sqrt{1 - (\sin\theta)^2} \cos\theta \, d\theta$$

$$= \frac{3^2}{5} \int_0^{\pi/2} (\cos\theta)^2 \, d\theta$$

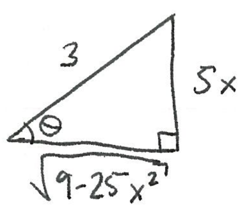
$$= \frac{3^2}{5} \int_0^{\pi/2} \left(\frac{1 + \cos(2\theta)}{2}\right) d\theta$$

$$= \frac{3^2}{5} \left(\frac{1}{2}\theta + \frac{1}{4}\cos(2\theta)\right)\Big|_0^{\pi/2}$$

$$= \frac{9}{10} \left(\theta + \frac{1}{2}\cos(2\theta)\right)\Big|_0^{\pi/2}$$

$$= \frac{9}{10} \left(\left(\frac{\pi}{2} + \frac{1}{2}\cos(\pi)\right) - \left(0 + \frac{1}{2}\cos(0)\right)\right)$$

$$= \frac{9}{10} \left(\frac{\pi}{2} + \frac{-1}{2} - \frac{1}{2}\right) = \boxed{\frac{9}{10}\left(\pi/2 - 1\right)}$$



$\sin(\theta) = \frac{5x}{3}$

$\frac{3\sin\theta}{5} = x$

$dx = \frac{3}{5}\cos\theta \, d\theta$

$x = 0 \longrightarrow \sin\theta = 0 \longrightarrow \theta = 0$

$x = \frac{3}{5} \longrightarrow \sin\theta = 1 \longrightarrow \theta = \frac{\pi}{2}$

2. Consider $I = \int_0^3 \left(\frac{1}{2}\right)^x dx$

(a) Estimate with $n=6$ using Midpoint Rule:

$\Delta x = \frac{3-0}{6} = \frac{1}{2}$, $f(x) = \left(\frac{1}{2}\right)^x$

$$S_M = \Delta x \left(f\left(\tfrac{1}{4}\right) + f\left(\tfrac{3}{4}\right) + f\left(\tfrac{5}{4}\right) + f\left(\tfrac{7}{4}\right) + f\left(\tfrac{9}{4}\right) + f\left(\tfrac{11}{4}\right) \right)$$

$$= \frac{1}{2}\left(\left(\tfrac{1}{2}\right)^{1/4} + \left(\tfrac{1}{2}\right)^{3/4} + \left(\tfrac{1}{2}\right)^{5/4} + \left(\tfrac{1}{2}\right)^{7/4} + \left(\tfrac{1}{2}\right)^{9/4} + \left(\tfrac{1}{2}\right)^{11/4} \right)$$

(b) Error bound

$|E_M| \le \frac{K(b-a)^3}{24n^2}$, $K \ge |f''(x)|$ on $[a,b]$

$$f''(x) = \left(\left(\tfrac{1}{2}\right)^x\right)'' = \left(-\ln(2)\left(\tfrac{1}{2}\right)^x\right)'$$

$$= \underbrace{(\ln(2))^2 \left(\tfrac{1}{2}\right)^x}_{\text{decreasing on } [0,3]}$$

so abs max occurs at 0.

So we choose $K = (\ln(2))^2 \ge |f''(x)|$ on $[0,3]$

$$|E_M| \le \frac{(\ln(2))^2 (3-0)^3}{24 \cdot 6^2} = \frac{(\ln(2))^2\, 3^3}{24 \cdot 36}$$

(c) $|E_M| \le \frac{(\ln(2))^2\, 3^3}{24\, n^2} < 0.01$

$$\frac{(\ln(2))^2\, 3^2}{8\, n^2} < \frac{1}{100}$$

$$\sqrt{\frac{(\ln(2))^2 \cdot 9 \cdot 100}{8}} < n$$

$\approx$ 7.35194

So $n = 8$.

(a) Estimate with $n=6$ using Trapezoidal Rule:

$\Delta x = \frac{3-0}{6} = \frac{1}{2}$, $f(x) = \left(\frac{1}{2}\right)^x$

$$S_T = \frac{\Delta x}{2}\left(f(0) + 2f\left(\tfrac{1}{2}\right) + 2f(1) + 2f\left(\tfrac{3}{2}\right) + 2f(2) + 2f\left(\tfrac{5}{2}\right) + f(3) \right)$$

$$= \frac{1}{4}\left(\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{2}\right)^{1/2} + 2\left(\tfrac{1}{2}\right)^{1} + 2\left(\tfrac{1}{2}\right)^{3/2} + 2\left(\tfrac{1}{2}\right)^{2} + 2\left(\tfrac{1}{2}\right)^{5/2} + \left(\tfrac{1}{2}\right)^{3} \right)$$

(b) Error bound

$|E_T| \le \frac{K(b-a)^3}{12n^2}$, $K \ge |f''(x)|$ on $[a,b]$

$$f''(x) = (\ln(2))^2 \left(\tfrac{1}{2}\right)^x$$

Choose $K = (\ln(2))^2$

$$|E_T| \le \frac{(\ln(2))^2 (3-0)^3}{12 \cdot 6^2} = \frac{(\ln(2))^2\, 3^3}{12 \cdot 36}$$

(c) $|E_T| \le \frac{(\ln(2))^2\, 3^3}{12 \cdot n^2} < 0.01$

$$\frac{(\ln(2))^2 \cdot 9}{4\, n^2} < \frac{1}{100}$$

$$\sqrt{\frac{(\ln(2))^2 \cdot 9 \cdot 100}{4}} < n$$

$\approx$ 10.3972

So $n = 11$.

3. Determine if the integrals converge or diverge.

(a) $\int_0^{\pi/2} \tan(x)\,dx$

$$\int_0^{\pi/2} \tan(x)\,dx = \lim_{t\to\frac{\pi}{2}^-} \int_0^t \tan(x)\,dx$$

$$= \lim_{t\to\frac{\pi}{2}^-} \ln|\sec(x)|\Big|_0^t$$

$$= \lim_{t\to\frac{\pi}{2}^-} \ln|\sec(t)| - \ln|\sec(0)|$$

$$= \infty$$

So $\int_0^{\pi/2} \tan(x)\,dx$ diverges

(b) $\int_0^{\infty} y e^{-y}\,dy$

$$\int y e^{-y}\,dy = -ye^{-y} + \int e^{-y}\,dy = -ye^{-y} - e^{-y} + C$$

$u = y \qquad v = -e^{-y}$

$du = dy \qquad dv = e^{-y}\,dy$

$$\int_0^{\infty} ye^{-y} = -ye^{-y} - e^{-y}\Big|_0^{\infty} = \lim_{t\to\infty} \left(-ye^{-y} - e^{-y}\right)\Big|_0^t$$

$$= \lim_{t\to\infty} \left(\frac{-t}{e^t} - \frac{1}{e^t}\right) - (0-1)$$

$$= 1.$$

So $\int_0^{\infty} ye^{-y}\,dy = 1.$ (converges)

(c) $\displaystyle\int_5^\infty \frac{2x}{\sqrt[5]{x^7+x^2+x+1}}\,dx.$

Let $f(x) = \dfrac{2x}{\sqrt[5]{x^7+x^2+x+1}}$, $g(x) = \dfrac{1}{x^{2/5}}$. Continuous on $[5, \infty)$.

$\displaystyle\int_5^\infty g(x)\,dx = \int_5^\infty \frac{1}{x^{2/5}}\,dx$ diverges via p-test ($p = 2/5 \leq 1$).

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{2x/\sqrt[5]{x^7+x^2+x+1}}{1/x^{2/5}} = \lim_{x\to\infty} \frac{2x \cdot x^{2/5}}{\sqrt[5]{x^7+x^2+x+1}} = 2 > 0$$

finite

By the Limit Comparison Test, $\displaystyle\int_5^\infty \frac{1}{x^{2/5}}\,dx$ diverges.

(d) $\displaystyle\int_0^1 \ln(x)\,dx$

$$\int \ln(x)\,dx = \frac{1}{x} + C$$

$$\int_0^1 \ln(x)\,dx = \lim_{t\to 0^+} \frac{1}{x}\Big|_t^1$$

$$= \lim_{t\to 0^+} 1 - \frac{1}{t}$$

$$= -\infty$$

So $\displaystyle\int_0^1 \ln(x)\,dx$ diverges.

(e) $\displaystyle\int_1^\infty \frac{1}{2+\sin(x)+e^{x^2}}\,dx$

on $[1,\infty)$: $\displaystyle \underbrace{\frac{1}{2+\sin(x)+e^{x^2}}}_{= f(x)} \le \frac{1}{1+e^{x^2}} \le \frac{1}{e^{x^2}} \le \frac{1}{e^x} < \underbrace{\frac{1}{x^2}}_{= g(x)}$ $\quad f, g$ continuous on $[1,\infty)$

$\displaystyle\int_1^\infty g(x)\,dx = \int_1^\infty \frac{1}{x^2}\,dx$ converges by the p-test $(p=2>1)$.

By the comparison test, $\displaystyle\int_1^\infty \frac{1}{2+\sin(x)+e^{x^2}}\,dx$ converges.

(f) $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$

$$\int \frac{e^x}{1+e^{2x}}\,dx \overset{u=e^x,\ du=e^{2x}dx}{=} \int \frac{1}{1+u^2}\,du = \arctan(u)+C = \arctan(e^x)+C.$$

$$\int_0^\infty \frac{e^x}{1+e^{2x}}\,dx = \lim_{t\to\infty} \arctan(e^x)\Big|_0^t$$

$$= \lim_{t\to\infty} \arctan(e^t) - \arctan(1)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$\int_{-\infty}^0 \frac{e^x}{1+e^{2x}}\,dx = \lim_{t\to-\infty} \arctan(e^x)\Big|_t^0$$

$$= \lim_{t\to-\infty} \arctan(1) - \arctan(e^t)$$

$$= \frac{\pi}{4} + \frac{\pi}{2}$$

$$\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx = \int_{-\infty}^0 \frac{e^x}{1+e^{2x}}\,dx + \int_0^\infty \frac{e^x}{1+e^{2x}}\,dx = \left(\frac{\pi}{2} - \frac{\pi}{4}\right) + \left(\frac{\pi}{4} + \frac{\pi}{2}\right)$$

$= \pi$. (Converges)

(g) $\int_0^\infty \sin\left(\frac{x}{2}\right) dx$

$$\int_0^\infty \sin\left(\frac{x}{2}\right) dx = \lim_{t\to\infty} \int_0^t \sin\left(\frac{x}{2}\right) dx$$

$$= \lim_{t\to\infty} -2\cos\left(\frac{x}{2}\right) \Big|_0^t$$

$$= \lim_{t\to\infty} -2\cos\left(\frac{t}{2}\right) + 2\cos(0)$$ Does not exist!

So $\int_0^\infty \sin\left(\frac{x}{2}\right) dx$ diverges.

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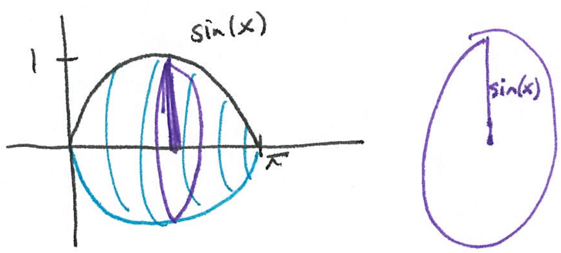
4. Consider the region R bounded by $f(x) = \sin(x)$ and the x-axis on $[0, \pi]$.

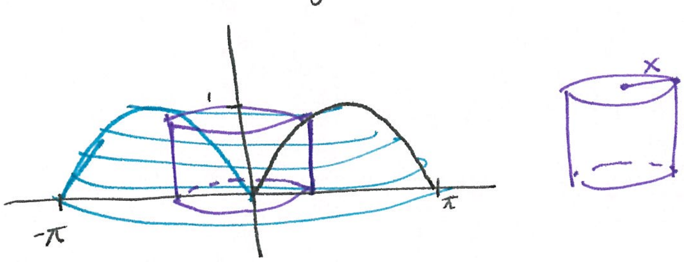
(a) Find the volume of the solid created by rotating the region R about the x-axis.



Area $= \pi(\sin(x))^2$

$$\boxed{V = \int_0^\pi \pi (\sin(x))^2\, dx}$$

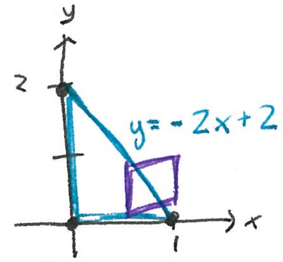
(b) Find the volume of the solid created by rotating the region R about the y-axis.

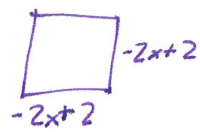


Area $= 2\pi x \sin(x)$

$$\boxed{V = \int_0^\pi 2\pi x \sin(x)\, dx}$$

5. Consider a 3-D solid S whose base is the trianglular region with vertices $(0,0)$, $(2,0)$, and $(0,1)$ and the cross-sections perpendicular to the x-axis are squares. Find the volume of S.





Area = $(-2x+2)^2$

$$V = \int_0^1 (-2x+2)^2\,dx$$

$$= \int_0^1 (4x^2 - 8x + 4)\,dx$$

$$= \left(\frac{4x^3}{3} - 4x^2 + 4x\right)\Big|_0^1$$

$$= \frac{4}{3} - 4 + 4$$

$$= \boxed{\frac{4}{3}}$$

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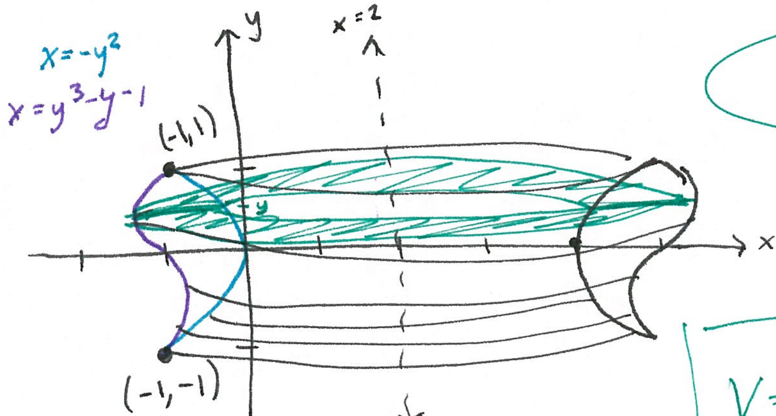
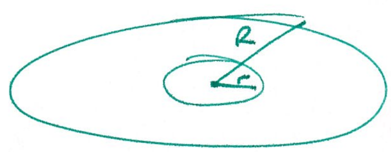
6. Consider the region A bounded by $x = y^3 - y - 1$ and $x = -y^2$. Set up but do not evaluate an integral that calculates the volume generated by rotating the region about the following lines.

(a) $x = 2$.





Area = $\pi(R^2 - r^2)$

$r = -y^2$

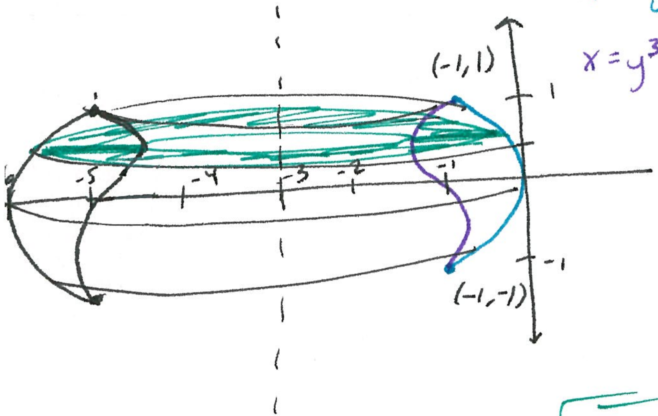
$R = (y^3 - y - 1)$

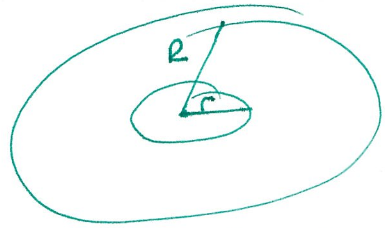
$$\boxed{V = \int_{-1}^{1} \pi\left((y^3-y-1)^2 - (-y^2)^2\right) dy}$$

(b) $x = -3$

$x=-y^2$

$x=y^3-y-1$





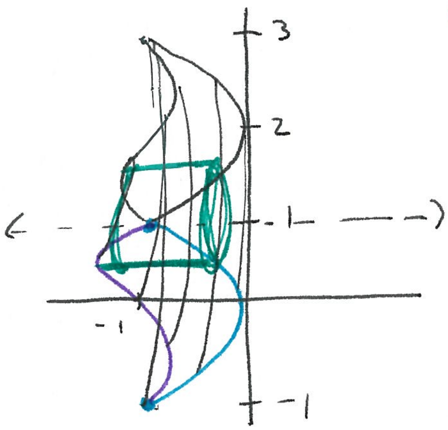
$\text{Area} = \pi(R^2 - r^2)$

$r = -3 - (y^3-y-1)$

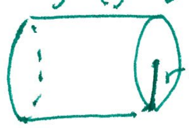
$R = -3 - (-y^2)$

$$\boxed{V = \int_{-1}^{1} \pi\left((-3+y^2)^2 - (-3-y^3+y+1)^2\right. dy}$$

(c) $y=1$



$h = -y^2-(y^3-y-1)$



$r = 1-y$

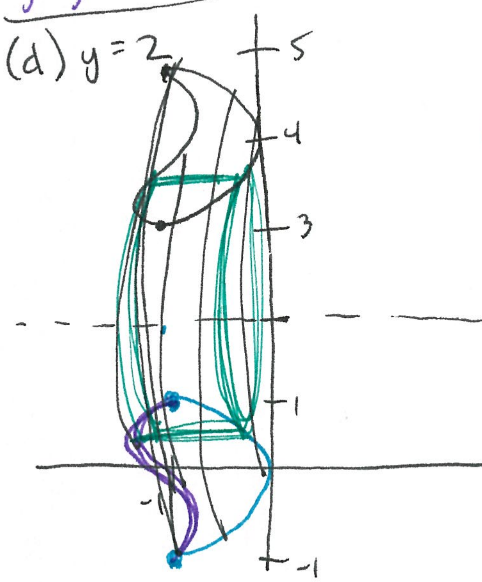
$\text{Area} = 2\pi r h$

$$\boxed{V = \int_{-1}^{1} 2\pi(1-y)\left(-y^2-(y^3-y-1)\right) dy}$$

$x=-y^2$

$x=y^3-y-1$

(d) $y=2$



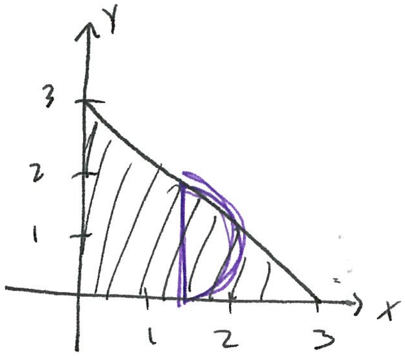
$h=-y^2-(y^3-y-1)$

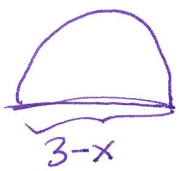
$r=2-y$

$\text{Area}=2\pi r h$

$$\boxed{V = \int_{-1}^{1} 2\pi(2-y)\left(-y^2-(y^3-y-1)\right) dy}$$

7. Consider a 3-D solid T whose base is the region bounded by $y=0$, $x=0$, and $y=3-x$ and whose cross-sections perpendicular to the x-axis are half discs. Find the volume of T.





$$\text{Area} = \frac{1}{2}\left(\pi\left(\frac{3-x}{2}\right)^2\right) = \frac{\pi}{2}\cdot\frac{(3-x)^2}{4} = \frac{\pi}{8}(3-x)^2$$

$$V = \int_0^3 \frac{\pi}{8}(3-x)^2\,dx$$

$$= \frac{\pi}{8}\int_0^3 9-6x+x^2\,dx$$

$$= \frac{\pi}{8}\left(9x - \frac{6x^2}{2} + \frac{x^3}{3}\right)\Bigg|_0^3$$

$$= \frac{\pi}{8}\left(9\cdot 3 - \frac{6\cdot 3^2}{2} + \frac{3^3}{3}\right)$$

$$= \frac{\pi}{8}(27 - 27 + 9)$$

$$= \boxed{\frac{9\pi}{8}}$$